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DCI-003-1016001

Seat No. ________

B. Sc. (Sem. VI) (CBCS) (W.E.F. 2016) Examination

July - 2022

Paper-8(A) : Graph Theory & Complex Analysis - II

Faculty Code : 003

Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all questions.

(2) Right hand side digits indicate the marks.

1 (a) Answer the following questions in short : **4**

(i) A complete graph with 7 vertices has __________ edge-disjoint Hamiltonian circuits.

(ii) Rank of a connected graph with n vertices and e edges is __________.

(iii) Define *isomorphic graphs.*

(iv) State the first theorem of graph theory.

(b) Attempt any **one** out of two : **2**

(i) If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vetices.

(ii) In any tree (with 2 or more vertices), prove that there are at least two pendent vertices.

(c) Attempt any **one** out of two : **3**

(i) Prove that in a binary tree with n vertices has $\frac{n+1}{2}$ pendent vertices.

(ii) Prove that the number of odd vertices in a graph are always even.

(d) Attempt any **one** out of two : **5**

(i) If a graph G with n vertices is a tree, then prove that it has $n-1$ edges. Discuss its converse with argument.

(ii) Prove that a given connected graph G is Euler if and only if all the vertices of G are of even degree.

2 (a) Answer the following questions in short : **4**

(i) Write adjacency matrix of complete graph with 5 vertices.

(ii) The chromatics number of binary tree with 11 vertices and of 5 level is ________.

(iii) Define : *acyclic graph.*

(iv) Define : *properly colored graph.*

(b) Attempt any **one** out of two : **2**

(i) In a standard notation fill in the blanks.

$n^* =$ ________, $f^* =$ ________, $r^* =$ ________,

$\mu^* =$ ________.

(ii) Prove that the reduced incidence matrix of a tree is non singular.

(c) Attempt any **one** out of two : **3**

(i) Prove that the ring sum of two circuits in a graph G is either a circuit or an edge-disjoint union of circuits.

(ii) Show that every circuit has an even number of edges in common with any cut-set.

(d) Attempt any **one** out of two : **5**

(i) With respect to a spanning tree T, prove that a branch b_i that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S, and no other.

(ii) Show that a connected planar graph with n vertices and e edges has $e-n+2$ regions.

3 (a) Answer the following questions in short : **4**

(i) The point that coincides with the transformation is called ________.

(ii) Define : *mobius mapping.*

(iii) Define : *conformal mapping.*

(iv) Define : *bilinear map.*

(b) Attempt any **one** out of two : **2**

(i) Find the fixed point of $w = \frac{z-1}{z+1}$.

(ii) Show that $x + y = 2$ transform into the parabola $u^2 = 8(v-2)$ under the transformation $w = z^2$.

(c) Attempt any **one** out of two : **3**

(i) Find bilinear transformation that maps –1, 0 and 1 into $-i$, 1 and i respectively.

(ii) Obtain a transformation of sector $r \le 1, 0 \le \theta \le \frac{\pi}{4}$ under the map $w = z^2$.

(d) Attempt any **one** out of two : **5**

(i) Prove that the transformation $(w+1)^2 = \frac{4}{z}$ transform the unit circle of w-plane into parabola of z-plane.

(ii) Show that the composition of two bilinear transformations is again a bilinear transformation.

4 (a) Answer the following questions in short : **4**

(i) Find the radius of convergence for the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

(ii) Define : Partial sum sequence for a complex series.

(iii) Write Maclaurian series expansion of $\sin z$.

(iv) State Laurent's theorem.

(b) Attempt any **one** out of two : **2**

(i) Find the limit of $\{z_n\}_{n=1}^{\infty}$, where $z_n = \frac{1}{n^3} + i$.

(ii) If a series of complex numbers converges, then show that the n^{th} term converges to zero as n tends to infinity.

(c) Attempt any **one** out of two : **3**

(i) Explain $\frac{z}{(z-2)(z-3)}$ in Laurent's series for $0 < |z-2| < 1$.

(ii) Show that $z \cosh z^2 = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n!)}$.

(d) Attempt any **one** out of two : **5**

(i) State and prove Taylor's theorem.

(ii) Expand $f(z) = \frac{1}{z^2 \sinh z}$ in Laurent's series for $z_0 = 0$ and hence deduce that $\int_C \frac{1}{\sinh z} dz = 2\pi i$ and $\int_C \frac{1}{z^2 \sinh z} dz = -\frac{\pi}{3} i$.

5 (a) Answer the following questions in short : **4**

(i) $\text{Res}\left(\frac{\cos z}{z}, 0\right)$ = ________.

(ii) Define : *Simple pole.*

(iii) Define : *Residue.*

(iv) Define : *Isolated singular point.*

(b) Attempt any **one** out of two : **2**

(i) Find the pole of $f(z) = \frac{e^{iz}(z+2)}{z^3 - 2z^2 - 5z + 6}$.

(ii) Find the residue of $f(z) = \frac{e^{3z}}{z(z-1)}$ at $z_0 = 0$.

(c) Attempt any **one** out of two : **3**

(i) If z_0 is the pole of order m for a complex valued function $f(z)$, then show that $\text{Res}(f(z), z_0) = \frac{\phi^{m-1}(z_0)}{(m-1)!}$, where $\phi(z) = (z - z_0)^m f(z)$.

(ii) Using residue theorem prove that

$$\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx = \frac{3\pi}{8}.$$

(d) Attempt any **one** out of two : **5**

(i) Evaluate $\int_{|z|=3} \frac{e^{tz}}{z^2(z^2+2z+2)} dz$.

(ii) State and prove Cauchy-Residue theorem.